CAPACITANCE

CONCEPT OF CAPACITANCE

Capacitance of a conductor is a measure of ability of the conductor to store charge on it. When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor. $Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacity of the conductor.

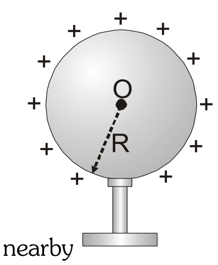
Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}\,L^{-2}\,T^4\,A^2$

Unit :- farad, coulomb/volt

The capacity of a conductor is independent of the charge given or its potential raised. It is also independent of nature of material and thickness of the conductor. Theoretically infinite amount of charge can be given to a conductor. But practically the electric field becomes so large that it causes ionisation of medium surrounding it. The charge on conductor leaks reducing its potential.

THE CAPACITANCE OF A SPHERICAL CONDUCTOR

When a charge Q is given to a isolated spherical conductor then its potential rises.



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

If conductor is placed in a medium then $C_{medium} = 4\pi\varepsilon R = 4\pi\varepsilon_0\varepsilon_r R$

Capacitance depends upon :

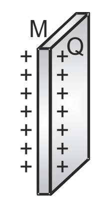
- Size and Shape of Conductor
- Surrounding medium
- Presence of other conductors nearby

CONDENSER/CAPACITOR

The pair of conductor of opposite charges on which sufficient quantity of charge may be accommodated is defined as condenser.

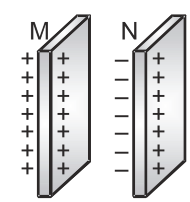
- **Principle of a Condenser**

It is based on the fact that capacitance can be increased by reducing potential keeping the charge constant.



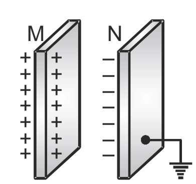
Consider a conducting plate M which is given a charge Q such that its potential rises to V then

$$C = \frac{Q}{V}$$



Let us place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure). If V_- is the potential at M due to induced negative charge on N and V_+ is the potential at M due to induced positive charge on N, then

$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$



Since V' < V (as the induced negative charge lies closer to the plate M in comparison to induced positive charge). $\Rightarrow$ C' > C Further, if N is earthed from the outer side (see figure) then $V'' = V_+ - V_-$ ($\because$ the entire positive charge flows to the earth)

$$C'' = \frac{Q}{V''} = \frac{Q}{V - V_-} \Rightarrow C'' >> C$$

If an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable. This is the principle of a parallel plate capacitor.

ENERGY STORED IN A CHARGED CONDUCTOR/CAPACITOR

Let C is capacitance of a conductor. On being connected to a battery. It charges to a potential V from zero potential. If q is charge on the conductor at that time then $q = CV$. Let battery supplies small amount of charge dq to the conductor at constant potential V. Then small amount of work done by the battery against the force exerted by exsiting charge is

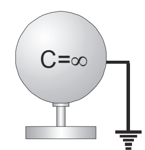
$$dW = Vdq = \frac{q}{C}dq \Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor. This work done is stored as potential energy, so

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV \qquad \therefore U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

GOLDEN KEY POINTS

- As the potential of the Earth is assumed to be zero, capacity of earth or a conductor connceted to earth will be infinite $C = \frac{q}{V} = \frac{q}{0} = \infty$

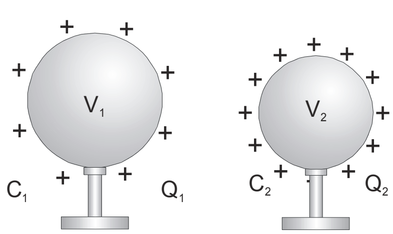


- Actual capacity of the Earth $C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}\times 64\times10^5 = 711\ \mu F$
- Work done by battery W_b = (charge given by battery) (emf) = QV but Energy stored in conductor $= \frac{1}{2}QV$ so 50% energy supplied by the battery is lost in form of heat.

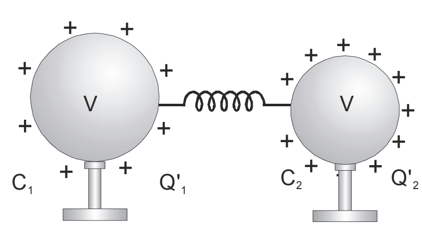
REDISTRIBUTION OF CHARGES AND LOSS OF ENERGY

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal.

Let the amounts of charges after the conductors are connected are Q_1' and Q_2' respectively and potential is V then



(Before connection)



(After connection)

- **Common potential**

According to law of Conservation of charge $Q_{\text{before connection}} = Q_{\text{after connection}} \Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$

Common potential after connection $\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$

- **Charges after connection**

$$Q_1' = C_1V = C_1\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_1}{C_1+C_2}\right)Q \qquad \text{(Q : Total charge on system)}$$

$$Q_2' = C_2V = C_2\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_2}{C_1+C_2}\right)Q$$

Ratio of the charges after redistribution $\boxed{\dfrac{Q_1'}{Q_2'} = \dfrac{C_1V}{C_2V} = \dfrac{R_1}{R_2} \text{ (in case of spherical conductors)}}$

- **Loss of energy in redistribution**

When charge flows through the conducting wire then **energy is lost mainly on account of Joule effect**, electrical energy is converted into heat energy, so change in energy of this system,

$$\Delta U = U_f - U_i \Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1-V_2)^2$$

Here negative sign indicates that energy of the system decreases in the process.

Example

A conductor gets a charge of 50 μC when it is connected to a battery of e.m.f. 5 V. Calculate capacity of the conductor.

Solution

Capacity of the conductor $C = \dfrac{Q}{V} = \dfrac{50\times10^{-6}}{5} = 10\mu F$

Example

The capacity of a spherical capacitor in air is 50 μF and on immersing it into oil it becomes 110 μF. Calculate the dielectric constant of oil.

Solution

Dielectric constant of oil $\varepsilon_r = \dfrac{C_{medium}}{C_{air}} = \dfrac{110}{50} = 2.2$

Example

A radio active source in the form of a metal sphere of diameter 10^{-3}m emits β particles at a constant rate of 6.25 10^{10} particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1.0 volt, assuming that 80% of emitted β particles escape from the surface.

Solution

Capacitance of sphere C $= 4\pi\varepsilon_0 R = \dfrac{0.5\times10^{-3}}{9\quad 10^{9}} = \dfrac{1}{18}\times10^{-12}F$

Rate to escape of charge from surface $= \dfrac{80}{100}\times6.25\times10^{10}\times1.6\times10^{-19}$ = 8 10^{-9} C/s

therefore q = (8 10^{-9}) t and q = CV $\Rightarrow$ 8 10^{-9} $t = \dfrac{1}{18}\times10^{-12}\times1 \Rightarrow t = \dfrac{10^{-12}}{8\times10^{-9}\times18} = \dfrac{10^{-3}}{144}$ = 6.95 μs

Example

The plates of a capacitor are charged to a potential difference of 100 V and then connected across a resister. The potential difference across the capacitor decays exponentially with respect to time. After one second the potential difference between the plates of the capacitor is 80 V. What is the fraction of the stored energy which has been dissipated ?

Solution

Energy losses $\Delta U = \frac{1}{2}CV_0^2 - \frac{1}{2}CV^2$

Fractional energy loss $\frac{\Delta U}{U_0} = \frac{\frac{1}{2}CV_0^2 - \frac{1}{2}CV^2}{\frac{1}{2}CV_0^2} = \frac{V_0^2 - V^2}{V_0^2} = \frac{(100)^2 - (80)^2}{(100)^2} = \frac{20 \times 180}{(100)^2} = \frac{9}{25}$

Example

Two uniformly charged spherical drops at potential V coalesce to form a larger drop. If capacity of each smaller drop is C then find capacity and potential of larger drop.

Solution

When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is radius and q is charge on smaller drop then $C = 4\pi\varepsilon_0 r$ and $q = CV$

Equating volume we get $\frac{4}{3}\pi R^3 = 2\,\frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$

Capacitance of larger drop $C' = 4\pi\varepsilon_0 R = 2^{1/3} C$

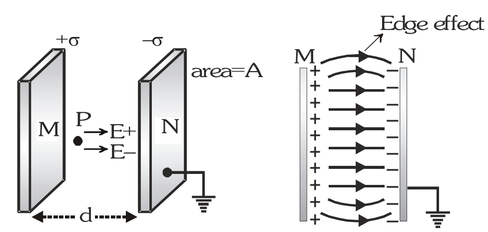
Charge on larger drop $Q = 2q = 2CV$

Potential of larger drop $V' = \frac{Q}{C'} = \frac{2CV}{2^{1/3}C} = 2^{2/3} V$

PARALLEL PLATE CAPACITOR

(i) Capacitance

It consists of two metallic plates M and N each of area A at separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then



I step : Finding electric field $E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$ [$\varepsilon = \varepsilon_0\,\varepsilon_r$]

II step : Finding potential difference $V = Ed = \frac{\sigma}{\varepsilon_0\varepsilon_r}d = \frac{qd}{A\varepsilon_0\varepsilon_r}$ ($\because E = \frac{V}{d}$ and $\sigma = \frac{q}{A}$)

III step : Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$

If medium between the plates is air or vacuum, then $\varepsilon_r = 1 \Rightarrow C_0 = \frac{\varepsilon_0 A}{d}$

so $C = \varepsilon_r C_0 = KC_0$ (where $\varepsilon_r = K$ = dielectric constant)

(ii) Force between the plates

The two plates of capacitor attract each other because they are oppositely charged.

Electric field due to positive plate $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{Q}{2\varepsilon_0 A}$

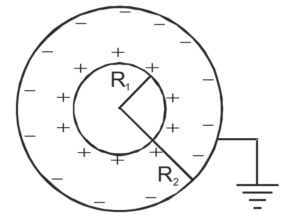
Force on negative charge –Q is $F = -Q\,E = -\dfrac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $F = \dfrac{Q^2}{2\varepsilon_0 A} = \dfrac{1}{2}\varepsilon_0 A E^2$

Force per unit area or energy density or electrostatic pressure $= \dfrac{F}{A} = u = p = \dfrac{1}{2}\epsilon_0 E^2$

SPHERICAL CAPACITOR

(i) Outer sphere is earthed



When a charge Q is given to inner sphere it is uniformly distributed on its surface A charge –Q is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.

$E = 0$ for $r < R_1$ and $E = 0$ for $r > R_2$

Potential of inner sphere $V_1 = \dfrac{Q}{4\pi\varepsilon_0 R_1} + \dfrac{-Q}{4\pi\varepsilon_0 R_2} \Rightarrow \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{R_2 - R_1}{R_1 R_2}\right)$

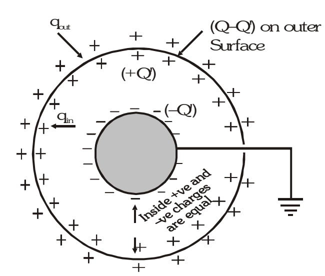
As outer surface is earthed so potential $V_2 = 0$

Potential difference between plates $V = V_1 - V_2 = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{(R_2 - R_1)}{R_1 R_2}$

So $C = \dfrac{Q}{V} = 4\pi\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1}$ (in air or vacuum)

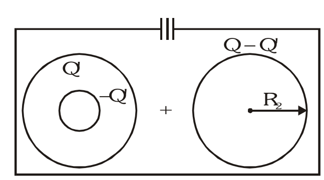
In presence of medium between plate $C = 4\pi\varepsilon_r\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1}$

(ii) Inner sphere is earthed



Here the system is equivalent to a spherical capacitor of inner and outer radii R_1 and R_2 respectively and a spherical conductor of radius R_2 in parallel. This is because charge Q given to outer sphere distributes in such a way that for the outer sphere.

Charge on the inner side is $Q' = \dfrac{R_1}{R_2} Q$ and

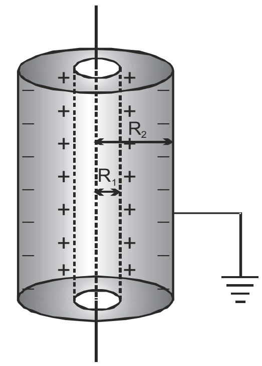


Charge on the outer side is $Q - \dfrac{R_1}{R_2} Q = \dfrac{(R_2 - R_1)}{R_2} Q$

So total capacity of the system. $C = 4\pi\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1} + 4\pi\varepsilon_0 R_2 = \dfrac{4\pi\varepsilon_0 R_2^2}{R_2 - R_1}$

CYLINDRICAL CAPACITOR

When a charge Q is given to inner cylinder it is uniformly distributed on its surface. A charge –Q is induced on inner surface of outer cylinder. The charge +Q induced on outer surface of outer cylinder flows to earth as it is grounded



Electrical field between cylinders $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{Q/\ell}{2\pi\varepsilon_0 r}$

Potential difference between plates $V = \displaystyle\int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0 r\ell}dr = \frac{Q}{2\pi\varepsilon_0\ell}\,\ell n\left(\frac{R_2}{R_1}\right)$

Capacitance $C = \dfrac{Q}{V} = \dfrac{2\pi\varepsilon_0\ell}{\log_e(R_2/R_1)}$

In presence of medium $C = \dfrac{2\pi\varepsilon_0\varepsilon_r\ell}{\log_e(R_2/R_1)}$

Example

The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Solution

The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0\left(\dfrac{ab}{b-a}\right)$

b = radius of the top of stratosphere layer = 6400 km + 50 km = 6450 km = 6.45 10^{6} m

a = radius of earth = 6400 km = 6.4 10^{6} m

$\therefore\ C = \dfrac{1}{9\times10^{9}}\times\dfrac{6.45\times10^{6}\times6.4\times10^{6}}{6.45\times10^{6}-6.4\times10^{6}}$ = 0.092 F

Example

A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder.

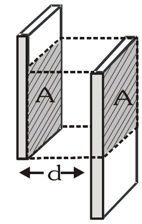
Solution

ℓ = 15 cm = 15 10^{-2} m; a = 1.4cm = 1.4 10^{-2} m ; b=1.5 cm = 1.5 10^{-2}m; q = 3.5 μC = 3.5 10^{-6}C

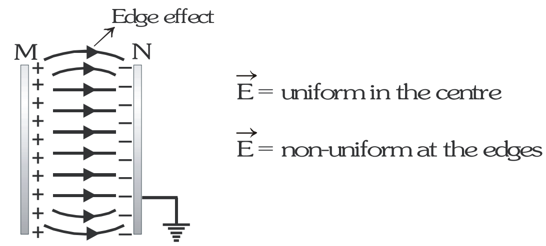
Capacitance $C = \dfrac{2\pi\varepsilon_0\ell}{2.303\log_{10}\left(\dfrac{b}{a}\right)} = \dfrac{2\pi\times8.854\times10^{-12}\times15\times10^{-2}}{2.303\log_{10}\dfrac{1.5\times10^{-2}}{1.4\times10^{-2}}}$ = 1.21 10^{-8} F

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them. Potential of inner cylinder, is $V = \dfrac{q}{C} = \dfrac{3.5\times10^{-6}}{1.2\times10^{-10}} = 2.89\times10^{4}\,V$

GOLDEN KEY POINTS

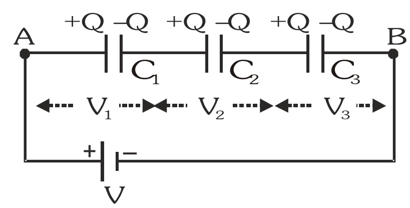


- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).
- If both the plates of parallel plate capacitor are touched each other resultant charge and potential became zero.
- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



COMBINATION OF CAPACITOR

- **Capacitor in series:**



In this arrangement of capacitors the charge has no alternative path(s) to flow.

(i) The charges on each capacitor are equal

i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

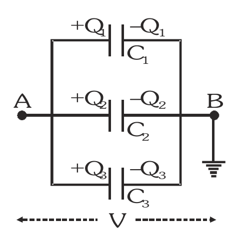
(ii) The total potential difference across AB is shared by the capacitors in the inverse ratio of the capacitances $V = V_1 + V_2 + V_3$

If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Capacitors in parallel**

In such in arrangement of capacitors the charge has an alternative path(s) to flow.



(i) The potential difference across each capacitor is same and equal the total potential applied. i.e. $V = V_1 = V_2 = V_3 \Rightarrow V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

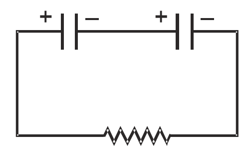
(ii) The total charge Q is shared by each capacitor in the direct ratio of the capacitances. $Q = Q_1 + Q_2 + Q_3$

If C_P is the net capacitance for the parallel combination of capacitors :

$$C_PV = C_1V + C_2V + C_3V \Rightarrow C_P = C_1 + C_2 + C_3$$

GOLDEN KEY POINTS

- For a given voltage to store maximum energy capacitors should be connected in parallel.
- If N identical capacitors each having breakdown voltage V are joined in
  (i) series then the break down voltage of the combination is equal to NV
  (ii) parallel then the breakdown voltage of the combination is equal to V.
- Two capacitors are connected in series with a battery. Now battery is removed and loose wires connected together then final charge on each capacitor is zero.



- If N identical capacitors are connected then $C_{series} = \dfrac{C}{N}$, $C_{parallel} = NC$
- In DC capacitor's offers infinite resistance in steady state, so there will be no current flows through capacitor branch.

Example

Capacitor C, 2C, 4C, ... ∞ are connected in parallel, then what will be their effective capacitance ?

Solution

Let the resultant capacitance be $C_{resultant}$ = C + 2C + 4C +... ∞ = C[1 +2 + 4 +... ∞] = C ∞ = ∞

Example

An infinite number of capacitors of capacitance C, 4C, 16C ... ∞ are connected in series then what will be their resultant capacitance ?

Solution

Let the equivalent capacitance of the combination = C_{eq}

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + \ldots \infty = \left[1 + \frac{1}{4} + \frac{1}{16} + \ldots \infty\right]\frac{1}{C} \quad \text{(this is G. P.series)}$$

$\Rightarrow S_\infty = \dfrac{a}{1-r}$ first term a = 1, common ratio r = $\dfrac{1}{4}$ $\Rightarrow \dfrac{1}{C_{eq}} = \dfrac{1}{1-\dfrac{1}{4}} \times \dfrac{1}{C} \Rightarrow C_{eq} = \dfrac{3}{4}C$

EFFECT OF DIELECTRIC

- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as **dielectrics**.
- Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.
- **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1L^1T^{-3}A^{-1}$

Polar dielectrics

- In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.
- In presence of external field dipoles tends to align in direction of field.
  Ex. Water, Alcohol, CO_2, $HC\ell$, NH_3

Non polar dielectrics

- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

  Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation :

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

- **Polarisation vector $\vec{P}$**

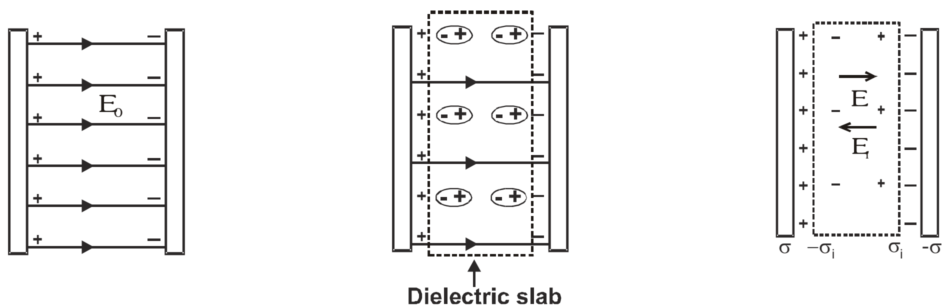
  This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

$\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

$\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i$ = induced surface charge density.

Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}T^1A^1$



Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{Q - Q_i}{\varepsilon_0} = \frac{V}{d}$

Capacitance in absence of dielectric $C_0 = \frac{Q}{V_0}$

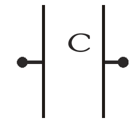
Capacitance in presence of dielectric $C = \frac{Q - Q_i}{V}$

The dielectric constant or relative permittivity K or $\varepsilon_r = \frac{E_0}{E} = \frac{V_0}{V} = \frac{C}{C_0} = \frac{Q}{Q - Q_i} = \frac{\sigma}{\sigma - \sigma_i} = \frac{\varepsilon}{\varepsilon_0}$

From $K = \frac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \frac{1}{K}\right)$ and $K = \frac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$

CAPACITY OF DIFFERENT CONFIGURATION

In case of parallel plate capacitor $C = \dfrac{\varepsilon_0 A}{d}$

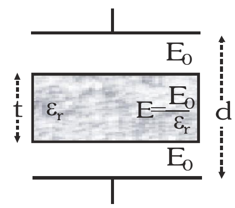


If capacitor is partially filled with dielectric

When the dielectric is filed partially between plates, the thickness of dielatric slab is t(t < d).

If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \dfrac{\sigma}{\varepsilon_0}$, exists in a space d.

On inserting the slab of thickness t, a field $E = \dfrac{E_0}{\varepsilon_r}$ exists inside the slab of thickness t and a field E_0 exists in remaining space (d – t). If V is total potential then $V = E_0(d - t) + E\,t$

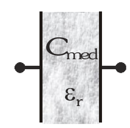


$$\Rightarrow V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \quad \because \quad \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] \Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} \quad ...(i)$$

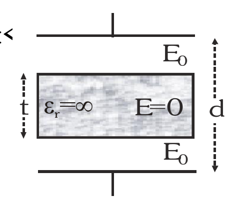
If medium is fully present between the space.

$\because t = d$



Now from equation (i) $\quad C_{medium} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

If capacitor is partialy filled by a conducting slab of thickness (t<



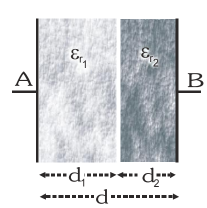
$\because \varepsilon_r = \infty$ for conductor $\quad C = \dfrac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} = \dfrac{\varepsilon_0 A}{(d - t)}$

DISTANCE AND AREA DIVISION BY DIELECTRIC

- **Distance Division**

(i) Distance is divided and area remains same.

(ii) Capacitors are in series.



(iii) Individual capacitances are $\quad C_1 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A}{d_1}, \; C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}$

These two in series $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \dfrac{d_2}{\varepsilon_0 \varepsilon_{r_2} A} \Rightarrow \dfrac{1}{C} = \dfrac{1}{\varepsilon_0 A}\left[\dfrac{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}{\varepsilon_{r_1} \varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A\left[\dfrac{\varepsilon_{r_1} \varepsilon_{r_2}}{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}\right]$

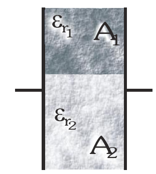
Special case : $\quad$ If $d_1 = d_2 = \dfrac{d}{2} \quad \Rightarrow \quad C = \dfrac{\varepsilon_0 A}{d}\left[\dfrac{2\varepsilon_{r_1} \varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$

- **Area Division**

(i) Area is divided and distance remains same.

(ii) Capacitors are in parallel.

(iii) Individual capacitances are $C_1 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A_1}{d}$ $C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$



These two in parallel so $C = C_1 + C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} + \dfrac{\varepsilon_0 \varepsilon_{r_2} A_2}{d} = \dfrac{\varepsilon_0}{d}(\varepsilon_{r_1} A_1 + \varepsilon_{r2} A_2)$

Special case : If $A_1 = A_2 = \dfrac{A}{2}$ Then $C = \dfrac{\varepsilon_0 A}{d}\left(\dfrac{\varepsilon_{r_1} + \varepsilon_{r2}}{2}\right)$

- **Variable Dielectric Constant :**

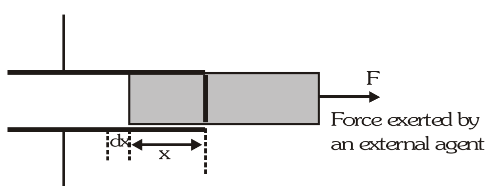
If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $C = \int dC$, where dC = capacitance of selected differential element.

(ii) If different element are in series, then $\dfrac{1}{C} = \int d\left(\dfrac{1}{C}\right)$ is solved to get equivalent capacitance C.

FORCE ON A DIELECTRIC IN A CAPACITOR

Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{Electrostatic} + W_F = 0$, where W_F denotes the work done by external agent in displacement dx

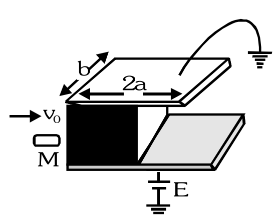


$W_F = -W_{Electrostatic}$ $W_F = \Delta U \Rightarrow -F.dx = \dfrac{Q^2}{2} d\left[\dfrac{1}{C}\right]$ $\left[W = \dfrac{Q^2}{2C}\right]$ $\Rightarrow -F.dx = \dfrac{-Q^2}{2C^2} dC \Rightarrow F = \dfrac{Q^2}{2C^2}\left(\dfrac{dC}{dx}\right)$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then as the p.d. across the plates is maintained constant. $V = \dfrac{Q}{C} \Rightarrow F = \dfrac{1}{2} V^2 \dfrac{dC}{dx}$.

Example

A parallel plate capacitor is half filled with a dielectric (K) of mass M. Capacitor is attached with a cell of emf E. Plates are held fixed on smooth insulating horizontal surface. A bullet of mass M hits the dielectric elastically and its found that dielectric just leaves out the capacitor. Find speed of bullet.



Solution

Since collision is elastic ∴ Velocity of dielectric after collision is v_0.

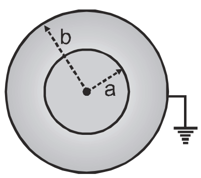
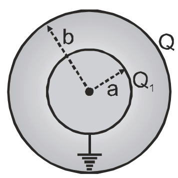
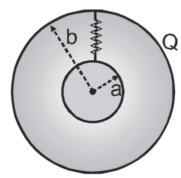
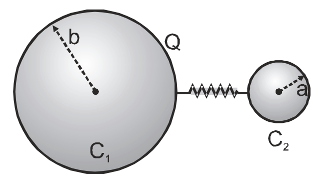
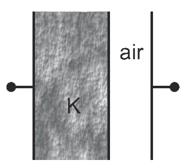
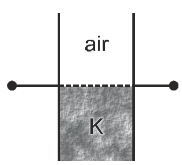
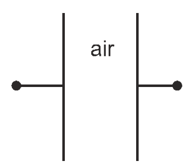
Dielectric will move and when it is coming out of capacitor a force is applied on

it by the capacitor $F = \frac{-dU}{dx} = \frac{-E^2\varepsilon_0 b(K-1)}{2d}$

Which decreases its speed to zero, till it comes out it travels a distance a.

$$\frac{1}{2}Mv_0^2 = \frac{E^2\varepsilon_0 b(K-1)a}{2d} \Rightarrow v_0 = E\left[\frac{\varepsilon_0 ab(K-1)}{Md}\right]^{1/2}$$

GOLDEN KEY POINTS

Spherical capacitor outer is earthed	Inner is earthed and outer is given a charge	Connected and outer is given a charge	Connected spheres
 $C = \frac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	 $C = \frac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	 $C = 4\pi\varepsilon_0 b$	 $C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a+b)$
 $C_1 = \left[\frac{2K}{K+1}\right]C$	 $C_2 = \left[\frac{K+1}{2}\right]C$	 $C_3 = C$ when no dielectric is used	
	$C_2 > C_1 > C_3$		

Example

A capacitor has two circular plates whose radius are 8cm and distance between them is 1mm. When mica (dielectric constant = 6) is placed between the plates, calculate the capacitance of this capacitor and the energy stored when it is given potential of 150 volt.

Solution

Area of plate $\pi r^2 = \pi\ (8\quad 10^{-2})^2 = 0.0201\ m^2$ and $d = 1mm = 1\quad 10^{-3}$ m

Capacity of capacitor $C = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{8.85\times10^{-12}\times6\times0.0201}{1\times10^{-3}} = 1.068\quad 10^{-9}$ F

Potential difference V = 150 volt

Energy stored $U = \frac{1}{2}CV^2 = \frac{1}{2}\times(1.068\times10^{-9})\times(150)^2 = 1.2\quad 10^{-5}$ J

Example

A parallel-plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9 10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

Solution

If the charge on the capacitor = Q

the surface charge density $\sigma = \frac{Q}{A}$ and the electric field = $\frac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9 10^7 V/m.

$\therefore$ if the maximum charge which can be given is Q then $\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7$ V/m

$\because$ A = 100 cm^2 = 10^{-2} m^2 $\Rightarrow$ Q = (5.0)   (10^{-2})   (8.85 10^{-12})   (1.9 10^7) = 8.4 10^{-6} C.

Example

The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3 10^4 V/m is established between the plates. It is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted into the (i) before the introduction of the metal plate and (ii) after its introduction. What would be the potential difference if a plate of dielectric constant K = 2 is introduced in place of metal plate ?

Solution

(i) In case of a capacitor as E = (V/d), the potential difference between the plates before the introduction of metal plate

$$V = E \quad d = 3 \quad 10^4 \quad 0.05 = 1.5 \text{ kV}$$

(ii) Now as after charging battery is removed , capacitor is isolated so q = constant. If C' and V' are the capacity and potential after the introduction of plate q = CV = C'V' i.e., $V' = \frac{C}{C'}V$

And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)}$, $V' = \frac{(d-t)+(t/K)}{d} \times V$

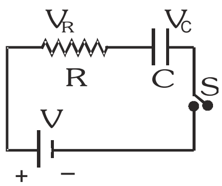
So in case of metal plate as K = ∞, $V_M = \left[\frac{d-t}{d}\right] \times V = \left[\frac{0.05-0.01}{0.05}\right] \times 1.5 = 1.2$ kV

And if instead of metal plate, dielectric with K = 2 is introduced $V_D = \left[\frac{(0.05-0.01)+(0001/2)}{0.05}\right] \times 1.5 = 1.35$ kV

CHARGING & DISCHARGING OF A CAPACITOR

Charging

- When a capacitor, resistance, battery, and key is conected in series and key is closed, then



- **Charge at any instant**

$$V = V_C + V_R = \frac{Q}{C} + IR \quad = \frac{Q}{C} + \frac{dQ}{dt}R$$

$$Q = CV\left[1 - e^{-t/RC}\right] = Q_0\left[1 - e^{-t/RC}\right]$$

At $t = \tau = RC$ = time constant

$Q = Q_0[1 - e^{-1}] = 0.632\ Q_0$

So, in charging, charge increases to 63.2% of charge in the time equal to τ.

- **Current at any instant**

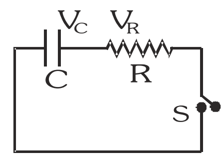
$i = dQ/dt \ = i_0 e^{-t/RC} \quad \{i_0 = Q_0/RC\}$

- **Potential at any instant**

$$V = V_0(1 - e^{-t/RC})$$

Discharging

- When a charged capacitor, resistance and keys is conected in series and key is closed. Then energy stored in capacitor is used to circulate current in the circuit.



- **Charge at any instant**

$V_C + V_R = 0$

$Q = Q_0 e^{-t/RC}$

At $t = \tau = RC$ = time constant

$Q = Q_0\, e^{-1} = 0.368\ Q_0$

So, in discharging, charge decreases to 36.8% of the initial charge in the time equal to τ.

- **Current at any instant**

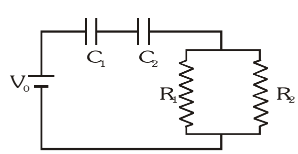
$i = dQ/dt = -i_0 e^{-t/RC} \quad \{i_0 = Q_0/RC\}$

- **Potential at any instant**

$$V = V_0\ e^{-t/RC}$$

Example

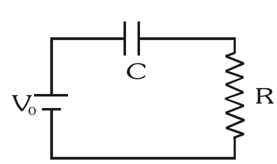
Find the time constant for given circuit if $R_1 = 4\Omega$, $R_2 = 12\Omega$, $C_1 = 3\mu F$ and $C_2 = 6\mu F$.



Solution

Given circuit can be reduced to : $C = \dfrac{C_1C_2}{C_1+C_2} = \dfrac{3\times 6}{3+6} = 2\mu F$, $R = \dfrac{R_1R_2}{R_1+R_2} = \dfrac{4\times 12}{4+12} = 3\Omega$

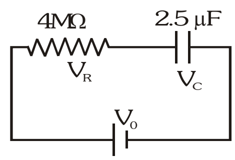
Time constant = RC = (3) (2 10^{-6}) = $6\mu s$



Example

A capacitor of 2.5 μF is charged through a series resistor of 4MΩ. In what time the potential drop across the the capacitor will become 3 times that of the resistor. (Given : $\ell n2 = 0.693$)

Solution



$V_C = V_0(1 - e^{-t/RC}) \because V_C = 3V_R \therefore V_0 = V_C + \frac{V_C}{3} \Rightarrow V_C = \frac{3}{4}V_0$

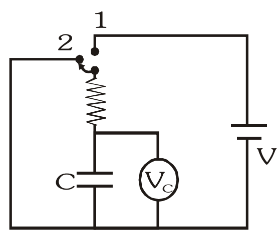
$\Rightarrow \frac{3}{4}V_0 = V_0(1 - e^{-t/RC}) \Rightarrow \frac{3}{4} = 1-e^{-t/RC} \Rightarrow \frac{1}{4} = e^{-t/RC} \Rightarrow 4 = e^{t/RC}$

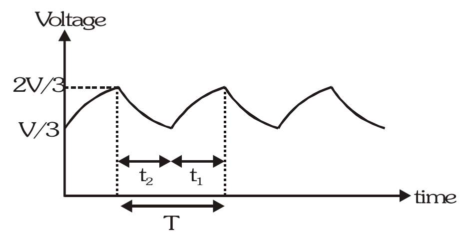
$\Rightarrow \frac{t}{RC} = \ell n4 \Rightarrow t = RC\,\ell n4 = 2RC\,\ell n2 = 2 \quad 4 \quad 10^6 \quad 2.5 \quad 10^{-6} \quad 0.693 = 13.86 \text{ s}$

SOME WORKED OUT EXAMPLES

Example#1

The switch in circuit shifts from 1 to 2 when $V_C > 2V/3$ and goes back to 1 from 2 when $V_C < V/3$. The voltmeter reads voltage as plotted. What is the period T of the wave form in terms of R and C?





(A) RC $\ell n3$ (B) 2RC ℓn 2 (C) $\frac{RC}{2}\ell n3$ (D) $\frac{RC}{3}\ell n3$

Solution **Ans. (B)**

During time 't_2' capacitor is discharging with the help of resistor 'R' $\therefore$ $q = q_0 e^{-t/RC}$

$V = V_0\ e^{-t/RC}$ [$\because$ Q=CV]

As $V_0 = \frac{2V}{3}$; $V = \frac{V}{3}$; $t_2 = RC\ \ell n2$

During time 't_1' capacitor is charging with the help of battery.

$\therefore$ $q = q_0\ (1-e^{-t/RC})$ or $V = V_0\ (1-e^{-t/RC})$

as $V_0 = \frac{2V}{3}$; $V = \frac{V}{3}$; $t_1 = RC\ \ell n2$

$T = t_1 + t_2 = 2RC\ \ell n2$

Example#2

Seven capacitors, each of capacitance 2μF are to be connected to obtain a capacitance of 10/11 μF. Which of the following combinations is possible ?

(A) 5 in parallel 2 in series (B) 4 in parallel 3 in series

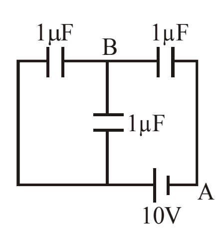
(C) 3 in parallel 4 in series (D) 2 in parallel 5 in series

Solution **Ans. (A)**

5(2μF) in series with $\left(\frac{2\mu F}{2}\right)$, 10μF in series with 1μF, $C_{eq} = \frac{10\times 1}{10+1} = \frac{10}{11}\mu F$

Example#3

In the circuit shown, if potential of A is 10V, then potential of B is -

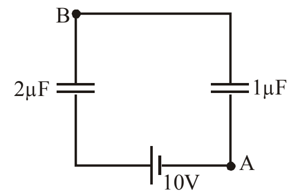


(A) 25/3 V (B) 50/3 V

(C) 100/3 V (D) 50 V

Solution **Ans. (B)**

Given circuit can be reduced as

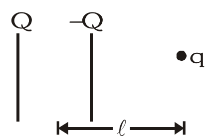


Charge on capacitors $=\left(\frac{2}{3}\right)(10)\mu C$

Now $V_B - V_A = \left(\frac{20}{3}\right)(1) = \frac{20}{3} \Rightarrow V_B = V_A + \frac{20}{3} = 10 + \frac{20}{3} = \frac{50}{3}V$

Example#4

The plates of very small size of a parallel plate capacitor are charged as shown .The force on the charged particle of charge 'q' at a distance 'ℓ' from the capacitor is : (Assume that the distance between the plates is $d << \ell$)



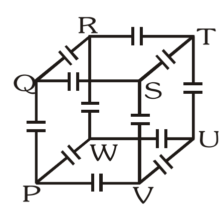
(A) Zero (B) $\frac{Qqd}{2\pi \in_0 \ell^3}$ (C) $\frac{Qqd}{\pi \in_0 \ell^3}$ (D) $\frac{Qqd}{4\pi \in_0 \ell^3}$

Solution **Ans. (B)**

Assume capacitor as dipole and use $F = qE$, $E = \frac{2kp}{r^3}$, $p = Qd$

Example#5

Twelve indentical capacitors each of capacitance C are connected as shown in figure.



(A) The effective capacitance between P and T is $\frac{6C}{5}$

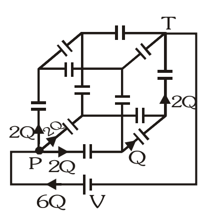
(B) The effective capacitance between P and U is $\frac{4C}{3}$

(C) The effective capacitance between P and V is $\frac{12C}{7}$

(D) All of the above statements are incorrect

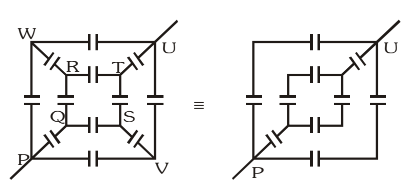
Solution **Ans. (A,B,C)**

For (A) :



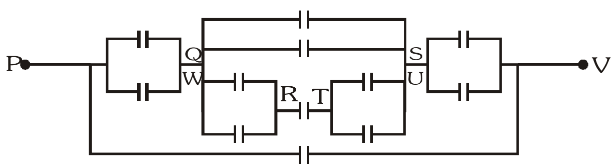
$V = \frac{2Q}{C} + \frac{Q}{C} + \frac{2Q}{C} = \frac{5Q}{C}$, $C_{eff} = \frac{6Q}{V} = \frac{6C}{5}$

For (B) : Given circuit can be drawn as



Equalvent capacitance between P and $U = \frac{C}{3} + \frac{C}{2} + \frac{C}{2} = \frac{4C}{3}$

For (C) :

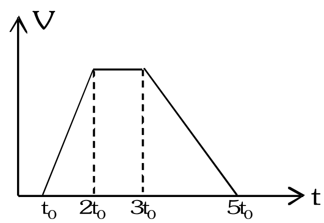


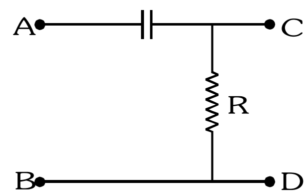
If a battery be connected across the terminals P and V, from symmetry $V_Q = V_W$ and $V_S = V_u$

$$\Rightarrow \text{Equivalent capacitance} = \frac{\left(\frac{5}{2}C\right)(C)}{\frac{5}{2}C + C} + C = \frac{12C}{7}$$

Example#6

A varying voltage is applied between the terminals A, B so that the voltage across the capacitor varies as shown in the figure Then.





(A) The voltage between the terminals C and D is constant between $2t_0$ and $3t_0$

(B) The current in the resistor is 0 between $2t_0$ and $3t_0$

(C) The current in the resistor between t_0 and $2t_0$ is twice the current between $3t_0$ and $5t_0$

(D) None of these

Solution **Ans. (ABCD)**

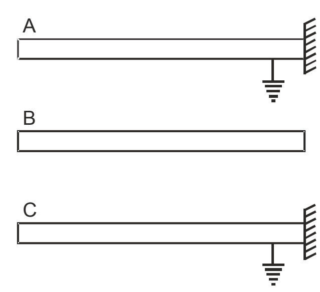
When the capacitor voltage is constant its charge is constant. No current in the resistor.

Also $C\frac{dV}{dt} = \frac{dq}{dt}$ is double between t_0 and $2t_0$ compared to $3t_0$ and $5t_0$

Example#7

A, B and C are three large, parallel conducting plates, placed horizontally. A and C are rigidly fixed and earthed. B is given some charge. Under electrostatic

and gravitational forces, B may be–



(A) in equilibrium midway between A and C.

(B) in equilibrium if it is closer to A than to C.

(C) in equilibrium if it is closer to C than to A.

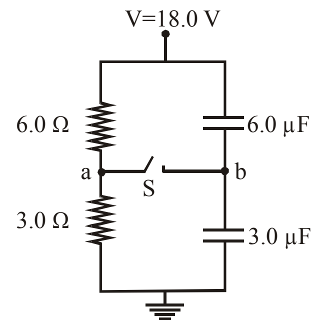
(D) B can never be in stable equilibrium.

Solution **Ans. (B, D)**

As A and C are earthed, they are connected to each other. Hence, 'A + B' and 'B + C' are two capacitors with the same potential difference. If B is closer to A than to C then the capacitance C_{AB} is > C_{BC}. The upper surface of B will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2, there will be a net upwards force on B. This can balance its weight.

Example#8

Study the following circuit diagram in figure and mark the correct option(s)



(A) The potential of point a with respect to point b when switch S is open is –6V.

(B) The points a and b, are at the same potential, when S is opened.

(C) The charge flows through switch S when it is closed is 54 µC

(D) The final potential of b with respect to ground when switch S is closed is 8V

Solution **Ans. (AC)**

When S is opened : $V_c - V_a = \dfrac{18 \times 6}{6+3} = 12V$

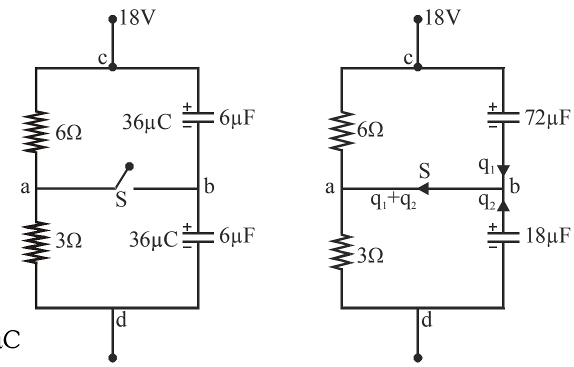
$V_c - V_b = \dfrac{18 \times 2}{6} = 6V \Rightarrow V_b - V_a = 12 - 6 = 6V$

Charges flown after S is closed :

$q_1 = 72 - 36 = 36\mu C$, $q_2 = 36 - 18 = 18\mu C$

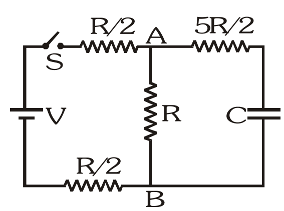
Charges flown through S after it is closed : 36 + 18 = 54 µC

Final potential of b is 6V



Example#9 to 11

In the circuit shown in figure, the battery is an ideal one with emf V. The capacitor is initially uncharged. The switch S is closed at time t = 0.



9. The charge Q on the capacitor at time t is-

(A) $\dfrac{CV}{2}\left(1 - e^{-\frac{t}{RC}}\right)$ (B) $\dfrac{CV}{2}\left(1 - e^{-\frac{t}{3RC}}\right)$ (C) $\dfrac{CV}{2}\left(1 - e^{-\frac{2t}{5RC}}\right)$ (D) $\dfrac{CV}{2}\left(1 - e^{-\frac{2t}{9RC}}\right)$

10. The current in AB at time t is-

(A) $\dfrac{V}{2R}\left(1 - e^{-\frac{t}{3RC}}\right)$ (B) $\dfrac{2V}{R}\left(1 - e^{-\frac{t}{3RC}}\right)$ (C) $\dfrac{2V}{R}\left(1 - \dfrac{e^{-\frac{t}{3RC}}}{6}\right)$ (D) $\dfrac{V}{2R}\left(1 - \dfrac{e^{-\frac{t}{3RC}}}{6}\right)$

11. What is its limiting value at $t \rightarrow \infty$?

(A) $\dfrac{V}{2R}$ (B) $\dfrac{V}{R}$ (C) $\dfrac{2V}{R}$ (D) $\dfrac{V}{3R}$

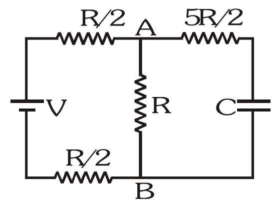
Solution

9. Ans. (B)

In steady state $V_C = V_{AB}$ = capacitor voltage = V/2

Calculation of time constant (τ_C)

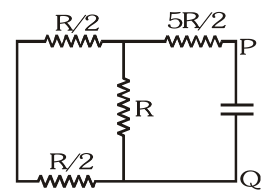
effective resistance across C = 3R



$$q = q_0\left(1 - e^{-\frac{t}{\tau_c}}\right),\ q_0 = C\frac{V}{2} \Rightarrow q = \frac{CV}{2}\left(1 - e^{\frac{t}{3RC}}\right)$$

10. Ans. (D)

$$V_{AB} = \frac{5}{2}Ri + \frac{q}{C}$$



$$\text{where } i = \frac{dq}{dt} = \frac{dv}{2 \times 3RC}e^{-\frac{t}{3RC}}, i = \frac{V}{6R}e^{-\frac{t}{3RC}}$$

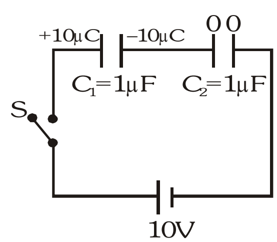
$$V_{AB} = \frac{5V}{12}e^{\frac{t}{3RC}} + \frac{V}{2}\left(1 - e^{-\frac{t}{3RC}}\right) \Rightarrow i_{AB} = \frac{V_{AB}}{R}$$

11. Ans. (A)

$$\text{At } t \to \infty\ V_{AB} = \frac{V}{2},\ i_{AB} = \frac{V}{2R}$$

Example#12 to 14

Following figure shows the initial charges on the capacitor. After the switch S is closed, find -



12. Charge on capacitor C_1

(A) 0 μC (B) 5 μC (C) 10 μC (D) None of these

13. Charge on capacitor C_2

(A) 0 μC (B) 5 μC (C) 10 μC (D) None of these

14. Work done by battery

(A) 50 μJ (B) 100 μJ (C) 150 μJ (D) None of these

Solution

12,13 Ans. (C), (A)

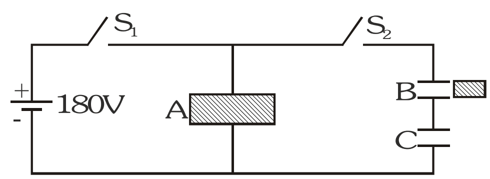
$10 - 2q + 10 = 0 \Rightarrow q=10$

14. Ans. (D)

$w_b = q_b(10) = 0$ charge flown through the battery is zero

Example#15 to 17

In the circuit shown, capacitor A has capacitance $C_1=2\mu F$ when filled with dielectric slab (k = 2). Capacitor B and C are air capacitors and have capacitances $C_2=3\mu F$ and $C_3=6\mu F$ respectively.



15. Calculate the energy supplied by battery during process of charging when switch S_1 is closed alone.

(A) 0.0324 J (B) 0.0648 J (C) 0.015 J (D) 0.030 J

16. Switch S_1 is opened and S_2 is closed . The charge on capacitor B is

(A) 240 μC (B) 280 μC (C) 180 μC (D) 200 μC

17. Now switch S_2 is opened, slab of A is removed. Another di–electric slab k = 2 which can just fill the space in B, is inserted into it and then switch S_2 is closed. The charge on capacitor B is

(A) 90 μC (B) 240 μC (C) 180 μC (D) 270 μC

Solution

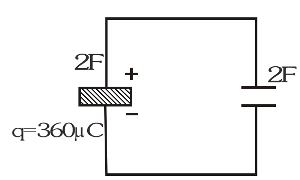
15. Ans. (B)

$q = CV = 2 \quad 10^{-6} \quad 180 = 360\ \mu C$

Energy supplied by battery = qV = 0.0648 J.

16. Ans (C)

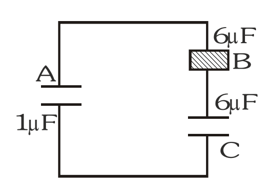
Equivalent of B & C = 2μF



Common potential $V = \dfrac{360\mu C}{4\mu F} = 90$ volt

$\therefore$ q on B = $90 \quad 2 \quad 10^{-6} = 180\ \mu C$.

17. Ans. (D)

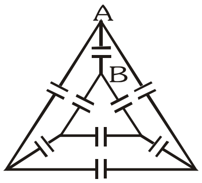
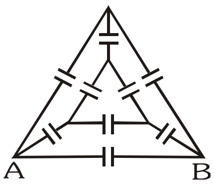
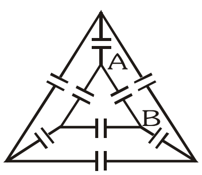
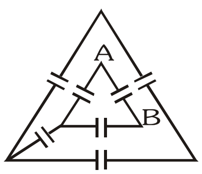


Common potential attained after S_2 is closed is $= \dfrac{360\mu C}{4\mu F} = 90$ volt.

$\therefore q_A = 90\ \mu C \quad \therefore q_B = 360\ \mu C - 90\ \mu C = 270\ \mu C$

Example#18

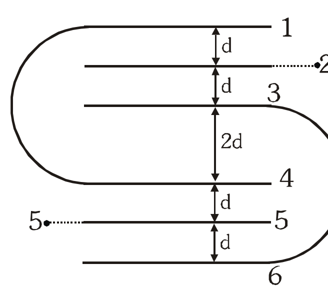
All capacitors given in column-I have capacitance of 1μF.

Column-I (Circuit)	Column-II (Capacitance)
(A) 	(P) $\frac{4}{3}\mu F$
(B) 	(Q) $\frac{3}{2}\mu F$
(C) 	(R) $\frac{15}{8}\mu F$
(D) 	(S) $\frac{5}{3}\mu F$
	(T) None of these

Solution **Ans.** (A)→(S), (B)→(R), (C)→(R), (D)→(Q)

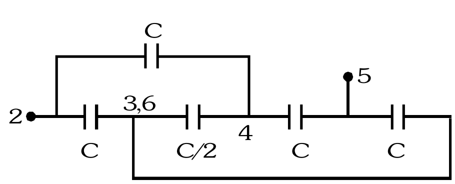
Example#19

There are six plates of equal area A and separation between the plates is d (d<<A) are arranged as shown in figure. The equivalent capacitance between points 2 and 5, is $\alpha \frac{\epsilon_0 A}{d}$. Then find the value of α.



Solution **Ans.** 1

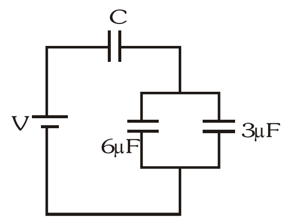
Redraw the circuit



$C_{eq} = C = \frac{\epsilon_0 A}{d}$

Example#20

If charge on 3μF capacitor is 3μC. Find the charge on capacitor of capacitance C in μC.



Solution **Ans. 9**

Potential difference across 3μF = P.D. across 6 μF = 1V

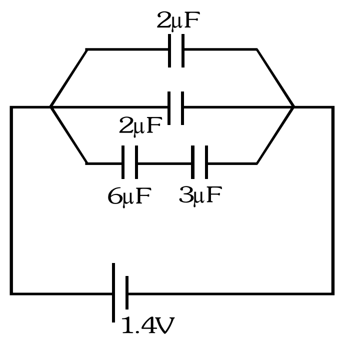
⇒ Charge on 6μF = 6μC

⇒ Total charge on combination of 6μF and 3μF = 9μC

Therefore charge on C = 9μC

Example#21

In the given circuit find energy stored in capacitors in mJ.



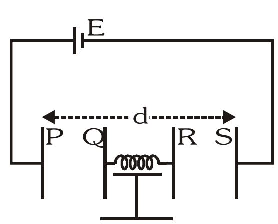
Solution **Ans. 6**

$C_{eq} = 2 + 2 + 2 = 6\ \mu F$

Energy stored = $\frac{1}{2}C_{eq}V^2 = \frac{1}{2}\left(6\times10^{-3}\right)(1.4)^2 = \left(3\times10^{-3}\right)(2) = 6\text{ mJ}$

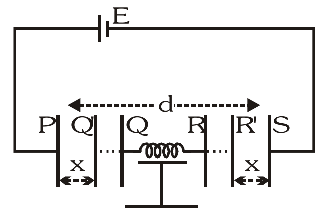
Example#22

Two parallel plate capacitors with area A are connected through a conducting spring of natural length ℓ in series as shown. Plates P and S have fixed positions at separation d. Now the plates are connected by a battery of emf E as shown. If the extension in the spring in equilibrium is equal to the separation between the plates, find the spring constant k.



Solution

Let charge on capacitors be q and separation between plates P and Q and R and S be x at any time distance between plates P and Q and R and S is same because force acting on them is same.



Capacitance of capacitor PQ, $C_1 = \dfrac{\varepsilon_0 A}{x}$

Capacitance of capacitor RS, $C_2 = \dfrac{\varepsilon_0 A}{x}$ From KVL $\dfrac{q}{C_1} + \dfrac{q}{C_2} = E \Rightarrow q = \dfrac{\varepsilon_0 AE}{2x}$

At this moment extension in spring, $y = d - 2x - \ell$.

Force on plate Q towards P, $F_1 = \dfrac{q^2}{2A\varepsilon_0} = \dfrac{\varepsilon_0^2 A^2 E^2}{8Ax^2\varepsilon_0} = \dfrac{A\varepsilon_0 E^2}{8x^2}$

Spring force on plate Q due to extension in spring, $F_2 = ky$

At equilibrium, separation between plates = extension in spring

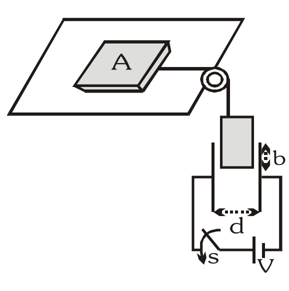
Thus $x = y = d - 2x - \ell \Rightarrow x = \dfrac{d-\ell}{3}$ ...(i) and $F_1 = F_2$ ...(ii)

From eq. (i) and (ii), $\dfrac{A\varepsilon_0 E^2}{8x^2} = ky = kx \Rightarrow x = \left(\dfrac{A\varepsilon_0 E^2}{8K}\right)^{1/3}$ ...(iii)

From eq. (i) and (iii), $\left(\dfrac{d-\ell}{3}\right) = \dfrac{A\varepsilon_0 E^2}{8K} \Rightarrow k = \dfrac{A\varepsilon_0 E^2 27}{8(d-\ell)^3}$

Example#23

A block A of mass m kept on a rough horizontal surface is connected to a dielectric slab of mass m/6 and dielectric constant K by means of a light and inextensible string passing over a fixed pulley as shown in fig. The dielectric can completely fill the space between the parallel plate capacitor of plate are ℓ ℓ and separation between the plates d kept in vertical position. Initially switch S is open and length of the dielectric inside the capacitor is b.



The coefficient of friction between the block A and the surface is $\dfrac{\mu}{4}$. Ignore any other friction.

(a) Find the minimum value of the emf V of the battery so that after closing the switch the block A will move

(b) If $V = 2V_{min}$ find the speed of the block A when the dielectric completely fills the space between the plates of the capacitor.

Solution

(a) The forces acting on the dielectric are electrostatic attractive force of field of capacitor and its weight. The block will slip when $F_E + mg \geq \mu Mg$ $F_E \geq \frac{M}{4}g - \frac{M}{6}g$

$$\frac{1}{2}\frac{\varepsilon_0 \ell}{d}(K-1)V^2 \geq \frac{Mg}{12} \quad \therefore V_{min} = \sqrt{\frac{Mg}{12} \times \frac{2d}{\varepsilon_0 \ell (K-1)}} = \sqrt{\frac{Mgd}{6\varepsilon_0 \ell (K-1)}}$$

(b) Now $V = 2V_{min}$. In this case the block will accelerate

Dielectric : $F_E + mg - T = ma$ ...(i) and Block : $T - \mu Mg = Ma$ ...(ii)

eq. (i) and (ii) give $a = \frac{F_E + (m - \mu M)g}{m + M}$ As $F_E = \frac{1}{2}\frac{\varepsilon_0 \ell}{d}(K-1)\ V^2 = \frac{1}{2}\frac{\varepsilon_0 \ell}{d}(K-1)\quad 4 \quad \frac{Mgd}{6\varepsilon_0 \ell (K-1)} = 2Mg$

$$\text{Thus } a = \frac{2Mg - \frac{M}{12}g}{\frac{7M}{6}} = \frac{23g \times 6}{7} = \frac{138}{7}g$$

From equation of motion, $v^2 = 2as \Rightarrow v^2 = 2\left(\frac{138g}{7}\right) \times (\ell - b) \Rightarrow v = \sqrt{\frac{276}{7}g(\ell - b)}$